On the Prospects of Explaining and Modeling with Higher Accuracy the Precession-nutation from VLBI Solutions

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Abstract We show the extent of the variance of CPO time series solutions from VLBI that can be reduced by adding suitable corrections to the precession model, and provide information concerning nutations.

Keywords VLBI, CPO, precession, standard models

1 Introduction

According to the IERS Conventions (2010) [1] the rotation relating to the terrestrial and celestial geocentric reference frames is characterized by five angles named the Earth orientation parameters (EOP). Among them, the deviations of the actual celestial intermediate pole (CIP) with respect to its location according to the conventional a priori precession-nutation model IAU2000A/2006 are the celestial pole offsets (CPO), usually expressed as the pair dX and dY. For decades, accurate enough CPO solutions can be only derived from Very Long Baseline Interferometry (VLBI) observations. Therefore, most of the actual CPO values directly inferred from observations comes from the analysis of the 24-hour-long VLBI R1 and R4 sessions, performed twice a week. Each IVS operational Analysis Center (AC) routinely computes solutions for the EOP on a session-wise basis with a latency of a few weeks, which is higher for the combined solutions derived by the IVS Combination Centers. In turn, the IERS Earth Orientation Center produces and releases a series of daily EOP, the CPO pair being time–densified by applying sophisticated algorithms.

Whatever the analysis procedure is, the obtaining of CPO solutions from VLBI data is impossible without using some a priori model for the location of the CIP, given the magnitude of its motion. The scatter of the CPO time series indicates how good the a priori is and would vanish if the latter was perfect. But this is not the case and it is known that the precession-nutation theories suffer from inaccuracies, inconsistencies, and have outdated components—and they are not the only EOP affected by issues (Ferrándiz et al., 2021 [2]). A prompt improvement of the Earth rotation theories and models was encouraged by Resolution 5 of the IAG (International Association of Geodesy) in 2019 and then by Resolution B2 of the IAU (International Astronomical Union) in 2021. The IAU/IAG Joint Working Group on Improving Theories and Models of the Earth's Rotation (JWG ITMER) is contributing to the implementation of the said resolutions. Matters related to the improvement of the current precession-nutation models are prioritized, according to the recommendations of the 2019 International Earth Rotation Service/Global Geodetic Observing System Unified Analysis Workshop. A replacement of the whole theories in force seems unfeasible at the short-term, because of its too high effectiveness-cost ratio; however, their improvement by supplementing each one with suitable corrections seems possible.

In this contribution we use a sample of single AC and combined CPO solutions, paying attention to the unexplained variance of those CPO time

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series—expressed as weighted root mean square (WRMS)—and present results on to which extent it could be reduced at short term by using suitable sets of corrections to the current conventional models. We focus mainly on the precession model, which may be improved quite easily and keeping its theoretical framework.

2 Precession

The IAU2006 precession model consists of the two components of the P03 theory (Capitaine et al., 2003 [3]), namely the precession of the equator and that of the ecliptic. Two main new features introduced by P03 are using 5-degree polynomials to model the two former components and considering the Earth's dynamical ellipticity H not as a constant like up to then, but as a linear function of time, taking into account the Earth's J_2 rate inferred at that time from the satellite laser ranging (SLR) observations available since 1976.

In the last years, different authors have brought to light some issues related to the two former features:

- Accuracy of the precession polynomials when testing them with the observations available so far;
- Inconsistencies between IAU2006 and the IAU2000 nutation model, where H is constant;
- Other issues arising from the J_2 time-variation actually inferred from observations versus the simplified model used in P03.

2.1 Linear Detrending

Regarding the first subject, the far dominant components of precession are the linear terms. The adopted values of the precession rates and offsets (at the initial date J2000.0) need some updating, according to all the analyses published in the last years (e.g., Belda et al. 2017 [4], Nurul et al. 2020 [5], Zhu et al. 2021 [6]). The magnitudes of the biases of the former parameters are small compared to the overall uncertainties, but statistically significant.

Therefore, a main objective is to estimate optimal values for them in an attempt to derive corrections that may reach a wide agreement and be applied as standards. We address that problem using exclusive VLBI observations. As a common requirement, all the results showed next are based on solutions that cover the period 1984–2021, have over 4,000 points, and provide CPO in terms of dX and dY. That interval was chosen because it spans two full cycles of the lunar node and we considered that convenient for analyzing nutations despite the poorer accuracy of the observations till the early 1990s; moreover, some experiments performed with the time interval starting in 1990 supported this idea.

This criterion led to select four recent solutions from single ACs, namely those identified with labels bkg2020a, gsf2020a, opa2020a, and usn2021c; and five quarterly combined solutions, namely bkg20q2, dgfi20q2, gsfc20q2, ivs20q2X, and ivs19q4X. The ivs19 series ends earlier, but we preferred to keep it because its trend was closer to those of other combined solutions than to that of ivs20. Data were downloaded from CDDIS or BKG data sites[1].

We first show the results of performing a simple linear detrending to each solution, for the period 1984–2021. The results for the single AC ones are displayed in the upper part of Table 1, whose columns display the offsets and rates, the formal uncertainties of these parameters as provided by the fit software, the number of points and the WRMS of raw and detrended data, as well as the WRMS improvement percentage. The WRMS decrease is evident at first glance in most cases; the lowest value is obtained by the usn2021 series (in bold), followed by gsf2020.

Results for the combined solutions appear in the lower part of Table 1. It can be noticed that the lowest WRMS for dY corresponds to ivs20q2 and is 146 µas, whereas for dX it happens to appear for ivs19q4 with 153 µas vs. 168 µas for ivs20q2. These distinguished labels and WRMS are in bold. A quick look to both tables is enough to reveal that combined solutions provide lower WRMS than single AC ones.

The rates of each CPO from the former solutions are gathered in Figure 1 for easier viewing and comparison. Each individual trend is displayed as a point surrounded by its 95% confidence interval (CI). Besides, we plot the medians of each group of trends as vertical lines, as well as the mean extended to all the solutions. For the comparison sake, a vertical dotted line displays the trends extracted similarly from IERS14C04.

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[1] `https://cddis.nasa.gov/` or `ftp://ivs.bkg.bund.de`

Table 1: Reduction of WRMS by simple linear detrending in the period 1984–2021 of four selected recent IVS individual and combined quarterly solutions. Offsets are in J2000.0; units µas, years.

CPO	solutions	Offset [µas]	trend [µas/y]	σ offset	σ trend	No. obs.	WRMS raw	WRMS detrend	WRMS % gain		Offset [µas]	trend [µas/y]	σ offset	σ trend	WRMS raw	WRMS detrend	WRMS % gain
dX	bkg2020a	44.65	6.253	4.03	0.287	5989	214.6	172.6	19.6	dY	-14.39	1.649	4.10	0.292	176.1	175.6	0.3
	gsf2020a	22.38	6.058	3.66	0.262	6431	196.0	166.1	15.2		-88.69	2.285	3.84	0.274	184.5	172.8	6.4
	opa2021a	-28.26	4.797	4.34	0.308	7027	211.1	205.8	2.5		-129.62	3.177	4.23	0.304	223.1	201.1	9.8
	usn2021c	16.64	5.402	3.73	0.272	5613	183.4	**160.8**			-82.22	1.907	3.89	0.283	176.6	**165.2**	6.5
	MEANS	13.85	5.628						12.4		-78.73	2.255					5.7
dX	bkg20q2	2.18	4.691	4.1	0.329	4209	177.8	167.3	5.9	dY	-84.03	2.328	4.36	0.347	186.2	174.9	6.0
	dgfi20q2	-1.46	4.653	4.4	0.343	4034	172.6	162.4	5.9		-85.88	2.804	4.66	0.36	179.0	168.8	5.7
	gsfc20q2	19.95	3.101	4.03	0.313	4229	176.9 1	67.6	5.3		-90.42	2.435	4.29	0.332	190.8	177.8	6.8
	ivs20q2X	-27.55	10.029	4.89	0.351	4266	205.0	168.0	18.1		-93.94	3.061	4.36	0.309	158.2	**145.9**	7.8
	ivs19q4X	16.37	5.147	4.38	0.337	4215	173.1	**152.9**	11.74		-83.56	0.277	4.66	0.357	174.0	**159.3**	8.4

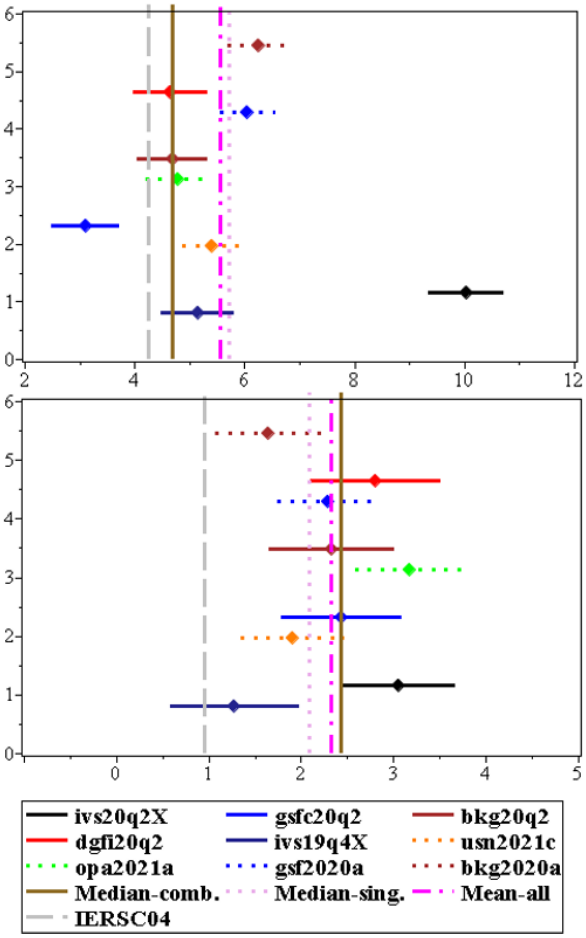

Fig. 1: CPO rates of individual and combined solutions (1984–2021) with CI 95% in µas: upper plot dX, lower plot dY.

Taking into account the former results and considerations, we think that the unexplained variance of the CPO may be reduced at short term in a noticeable but easy way by undertaking a few potential simple actions, e.g.:

- Determine, agree, and apply corrections to the dX and dY trends. Using the means of a sample of VLBI solutions or simply the trends of the IVS reference solution may be a rough first choice to start estimation and validation; for instance, reference initial values may be:
  - dX trend correction about 5.524 to 5.628 mas/yr
  - dY trend correction about 2.255 to 2.381 mas/yr
- Get more insight into the causes of the offsets—it might take longer

2.2 Inaccuracy Derived from Inconsistencies among IAU2006, IAU2000, and the Actual Variation of the Dynamical Ellipticity

The first of the last two issues cited at the beginning of this section arises from the fact that IAU2006 assumes a constant J_2 rate unlike IAU2000, which considers this precession parameter as a constant; moreover, both theories adopted different values for the obliquity and the "precession constant", i.e., the longitude rate proportional to the constant components of H (or J_2). These inconsistencies may be compensated by applying a few corrections to the nutation model derived by Escapa et al. (2017) [7]

$$\begin{aligned} \mathrm{dX} =& (-6.2+15.4\,t)\ \sin\Omega-(0.6+0.6\,t^2)\ \cos\Omega \\ &+1.4\,t\ \sin(2F-2D+2\Omega), \\ \mathrm{dY} =& (0.8-25.4\,t)\ \cos\Omega-(0.8+0.3\,t^2)\ \sin\Omega \\ &-(0.3+1.8\,t)\ \cos(2F-2D+2\Omega). \end{aligned}$$

The former corrections have small magnitude but contain secular–mixed terms whose amplitude grows with time and thus become more relevant as time gets away from the initial date J2000.0. For that reason, when the topic was discussed prior to the IAU 2018 General Assembly, it was decided that applying them was not urgent then and may be delayed until the time of adopting corrections of a higher magnitude.

The situation is worse when the actual J_2 time-variation is considered instead of the linear variation adopted by IAU2006, which was inferred from the Earth's oblateness trend of -3.001×10^{-8}/yr observed formerly. It is equivalent to a rate of $H_1 = H_0\ (-2.7719\times10^{-8})$/yr , H_0 being the reference dynamical ellipticity. The actual J_2 values were taken from the long-term solution provided by the Center for Space Research of The University of Texas at Austin (Cheng et al. 2013 [8]), shown in black in Figure 2, which also displays the linear model assumed by IAU2006 as a red line. However, the trend started to change sign in 1997, and has reversed along the last 20 years. At present, the observed $J_2 = -C_{20}$ evolution is better represented by a second-degree polynomial, depicted in blue in Figure 2 (see e.g., Marchenko & Lopushanskyi 2018 [9], Chao & Chung 2020 [10]).

By straightforward integration of the equations of motion, it is immediate that the linear H trend assumed in IAU2006 contributes a quadratic term to the precession polynomial in longitude, whereas a quadratic term of the fit contributes a cubic parabola to it. The effect of the observed H variations is more difficult to compute and thus the procedure cannot be detailed here; let us say that we used numerical methods under the Hamiltonian formalism. The longitude variations resulting from the observed values, expressed as dX, are shown in black in Figure 3 together with the foresaid quadratic and cubic components of dX, in red and blue respectively. It is clear that the cubic model for correcting the dX precession gives results closer to the actual dX variations computed numerically than the quadratic model currently in force.

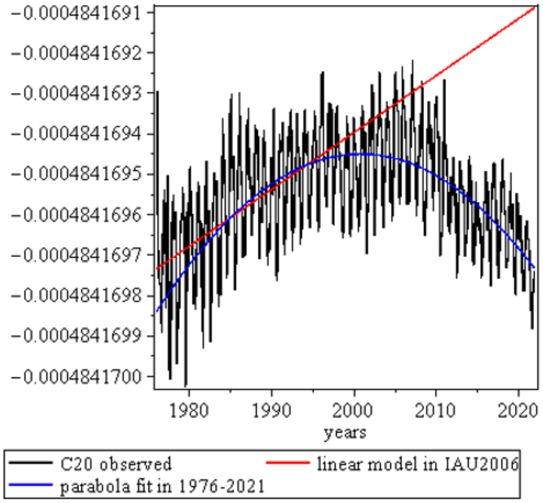

Fig. 2: C20 from CSR long-term series, observed (black), IAU2006 linear model (red), and fitted parabola (blue), 1976–2021.

Let us notice that such a quadratic component was not provided explicitly in P03 (Capitaine et al. 2003 [3]) and thus we would have to recompute it and then subtract it before applying the cubic correcting term if a second-degree polynomial would be adopted as the basic simple model for the H change.

Considering the H modeling related issues, we may think of different actions for easily improving the precession model at short term, like the following:

- Leave the theory as it is (no action) or update the value of the H linear trend;
- Replace the linear trend of H by a quadratic fit, a model closer to reality;
- Return to an *old-style* model with constant H.

3 Additional Comments and Conclusions

As for the forced nutation models, a first conclusion from practically all the analyses published in the last years is that the amplitudes of the main nutation components need also some updating (Belda et al. 2017 [4], Nurul et al. 2020 [5], Zhu et al. 2021 [6]). The WRMS may be reduced significantly (e.g., Ferrándiz et al. 2022 [11]).

The implementation of an agreed, suitable set of semi-empirical corrections to the precession and forced

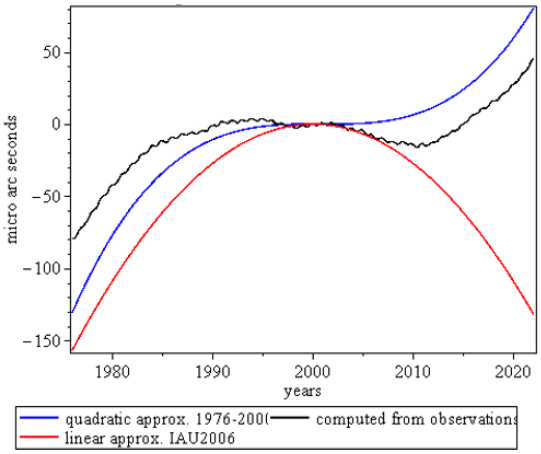

Fig. 3: dX variations from the observed J_2 (black), the IAU2006 linear approximation (red), and the parabolic fit (blue), 1979–2022.

nutation models IAU2006 and IAU2000 may not take long and bring the WRMS of each CPO to the vicinity of 120 µas for combined IVS solutions, or 140–150 µas for single analysis center solutions, respectively. Let us recall that significant further improvement of accuracy may be gained by using supplemental FCN models.

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