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"Baseline Length Repeatability"

Arthur Niell

Baseline Length Repeatability

A. Niell
MIT Haystack Observatory

1. Introduction

Baseline length is independent of rotations, so this makes it a good independent measure of the accuracy achieved for geodetic VLBI.

Assuming that the uncertainties are the same at both ends of the baseline, the horizontal and vertical station position uncertainties can be related to baseline length uncertainty (see Figure 1) as follows:

σ_v = vertical uncertainty

σ_h = horizontal uncertainty

L = baseline length

σ_L = baseline length uncertainty

r_E = Earth radius

From geometry

$$\frac{\sigma_{\mathrm{v}}}{\sigma_{L/2,\mathrm{v}}} = \frac{\mathrm{r_E}}{\mathrm{L}/2}$$

$$\sigma_{L/2,\mathrm{v}} = \frac{\mathrm{L}}{2\mathrm{r_E}}\sigma_{\mathrm{v}}$$

$$\frac{\sigma^2_{\mathrm{L/2,h}} - \sigma^2_{\mathrm{h}}}{\sigma^2_{\mathrm{L/2,h}}} = \left(\frac{\mathrm{r_E}}{\mathrm{L}/2}\right)^2 \tag{1.1}$$

$$\sigma^2_{\mathrm{L/2,h}} = \left(1 - \left(\frac{\mathrm{L}}{2\mathrm{r_E}}\right)^2\right)\sigma^2_{\mathrm{h}}$$

Add the horizontal and vertical uncertainties quadratically, and, assuming that the errors are independent at the two sites forming the baseline, multiply the variance by 2 to get the length uncertainty, in terms of the site local uncertainties.

$$\sigma^2_{\mathrm{L}} = 2\left(\sigma^2_{\mathrm{h}} + (\sigma^2_{\mathrm{v}} - \sigma^2_{\mathrm{h}})\left(\frac{\mathrm{L}}{2\mathrm{r_E}}\right)^2\right) \tag{1.2}$$

Baseline length repeatabilities can be used to estimate the effective vertical and horizontal uncertainties by using the weighted RMS baseline lengths, $R_{L,}$ as a measure of σ_L. Then by fitting for *a* and *b* (must be done iteratively) to

$$\mathrm{R}_{\mathrm{L}} = \left(a^2 + b^2 \cdot L^2\right)^{1/2} \tag{1.3}$$

the values for σ_h and σ_v can be calculated from the estimates of *a* and *b* as

$$\begin{aligned} \sigma_h &= \sqrt{a^2/2} \\ \sigma_v &= \sqrt{a^2/2 + 2r_E^2 b^2} \end{aligned} \tag{1.4}$$

This estimation is described in Appendix A and implemented in *plot_baseline_rl_cont05.m* in directory *c:\aa\cont\cont05*. The result for CONT05 baseline length uncertainties is shown in Figure 2, for which the weighting is the square of the baseline length uncertainty in order to weight the best determined baselines more. Furthermore, the local errors cannot add up to greater than the actual formal error.

2. WRMS, formal uncertainty, and unmodeled local error

The WRMS for a series of baseline measurements consists of the session baseline lerngth errors, σ_B, and other unmodeled error. In order to estimate the contribution of the additional error as equivalent topocentric errors, consider the WRMS as a quadratic sum of the formal error and local errors.

$$WRMS^2 = \sigma_{\mathrm{L}}^2 + 2\left(\sigma_{\mathrm{h}}^2 + (\sigma_{\mathrm{v}}^2 - \sigma_{\mathrm{h}}^2)\left(\frac{\mathrm{L}}{2\mathrm{r}_{\mathrm{E}}}\right)^2\right) \tag{2.1}$$

The unmodeled error, $WRMS^2 - \sigma_L^2$, can then be estimated as above.

Appendix A

Use non-linear weighted least-squares to estimate parameters *a* and *b* in

$$\mathrm{R} = \left(a^2 + b^2 \cdot L^2\right)^{1/2} \tag{A.1}$$

Let $$x = [a \;\; b]' \tag{A.2}$$

Use initial estimates $x_1 = [a_1 \;\; b_1]'$ of 2 mm, 5mm/10,000km from plot of CONT05 length uncertainties, then estimate corrections to them for $j = 1, n\_iter$.

$$\Delta R_j = R_{obs}(L) - R_j(L) \tag{A.3}$$

where $$R_j = \left(a_j^2 + b_j^2 \cdot L^2\right)^{1/2} \tag{A.4}$$

and R_{obs} are either the observed WRMS or the baseline length uncertainties.

$$\begin{aligned} \Delta R_j &= \mathbf{A}_j \Delta x_j \\ \mathbf{A}_j &= \left[\frac{\partial R_j(L)}{\partial a} \;\; \frac{\partial R_j(L)}{\partial b}\right] \\ \Delta x_j &= [\Delta a \;\; \Delta b]' \end{aligned} \tag{A.5}$$

$$\frac{\partial R_j(L)}{\partial a} = \frac{a_j}{R_j(L)}$$
$$\frac{\partial R_j(L)}{\partial b} = \frac{b_j}{R_j(L)} L^2 \quad \text{(A.6)}$$

So
$$\Delta x_j = \left(\mathbf{A}_j^{\mathrm{T}} \mathrm{W} \mathbf{A}_j \right)^{-1} \mathbf{A}_j^{\mathrm{T}} \mathrm{W} \Delta R_j \quad \text{(A.7)}$$

For weight, W, use the value of R_{obs}. Then
$$x_{j+1} = x_j + \Delta x_j \quad \text{(A.8)}$$

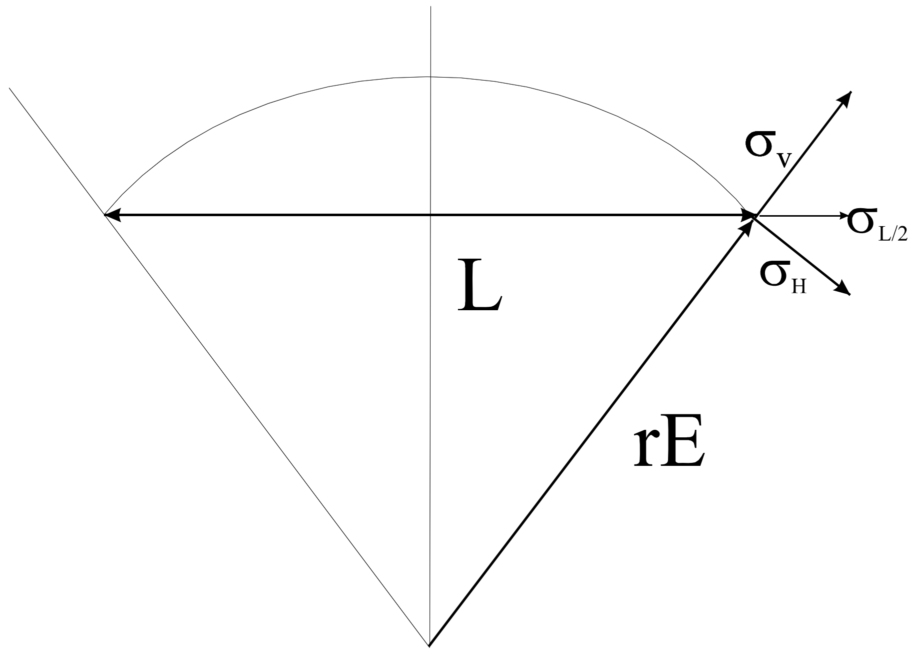

Figure 1. Geometry for relating vertical and horizontal site position errors to baseline length error. (*c:\aa\memos\baseline_repeatability_0.cdr*)

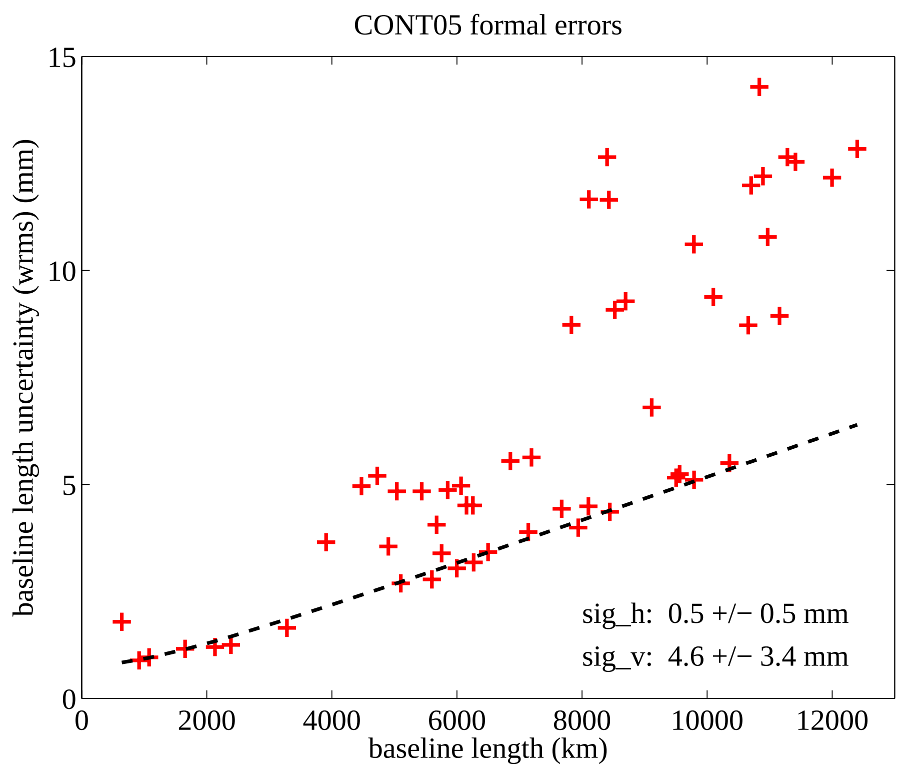

Figure 2. Least squares estimate of the local horizontal and vertical uncertainties that would provide the baseline length uncertainties for the CONT05 session C0501. (*c:\aa\cont\cont05\cont05_baseline_error.ps*)