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“Single Station Troposphere Delay Modeling”

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Memo: Single Station Troposphere Delay Modeling
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1. Introduction

This memo presents basic investigations on the behavior of wzd residuals and the performance of state-of-the-art wzd modeling, some experiments on elevation dependent weighting functions and a few considerations concerning the discussion about increasing the cutoff angle.

I carried out simulation studies using time series of turbulent equivalent wet zenith delay (provided by Tobias Nilsson). The simulated wzd was multiplied by 1/sin(e) to obtain slant delays which were used as observations to estimate wzd parameters in a classical least squares adjustment.
For these investigations I did not use any VLBI analysis software but worked with a MATLAB programme. All computations were performed for single stations using the first 20 hours of data of the schedule stat16_12_3p5D0.

The behavior of wzd residuals and downweighting functions were investigated in Section 2 and 3. To be sure that the residuals are due to deficiencies in modeling, no station clock or white noise were added. As the same mapping function was used for the estimation as was for the computation of slant delays, there is also no error due to the mapping function.

Furthermore some cutoff angle tests were performed in Section 4. As wzd, station clock and station height are highly correlated, all three parameters were included in these investigations to deduce more realistic results. The stochastic variations of station clocks were simulated with a random walk corresponding to an ASD of $2 \cdot 10^{-15}$ @ 50 min and added to the slant wet delay (turbulent equivalent wet zenith delay multiplied by a mapping function) and a white noise of 6 ps for each station.

2. Bubble plots of wzd residuals

To investigate the spatial and temporal coherence of wet zenith delays provided by the turbulence model on one hand and the behavior of wzd residuals on the other, the wzd residuals were plotted versus azimuth and elevation for a specified time interval. By shifting the time interval until the whole data was included, a time history of wzd residuals was created. The wzd was modeled with piecewise linear functions estimated for time intervals of 10 minutes (no gradients were estimated in this case).
The time histories of wzd residuals can be downloaded as pdf files for all stations from mars.hg.tuwien.ac.at/~vlbi2010/stat16_12_3p5D0/Bubble_plots/. The bubble plots are available in cartesian and polar presentation. For the cartesian bubble plots, the wzd residuals were plotted for a time interval of 1 h which was shifted by 10 min to create the time history. For the polar plots, a 20 min interval of residuals was shifted by 20 min, meaning that there is no overlap between the single plots.

Figure 1 shows such plots for the stations Ny Alesund (left) and HartRAO (right) for time epoch 13 h. The size of the bubbles corresponds to the absolute value of the wzd residuals, the color indicates the sign (red: negative residuals, blue: positive residuals). Ny Alesund is located far in the North where we do not expect the troposphere to vary very strongly.

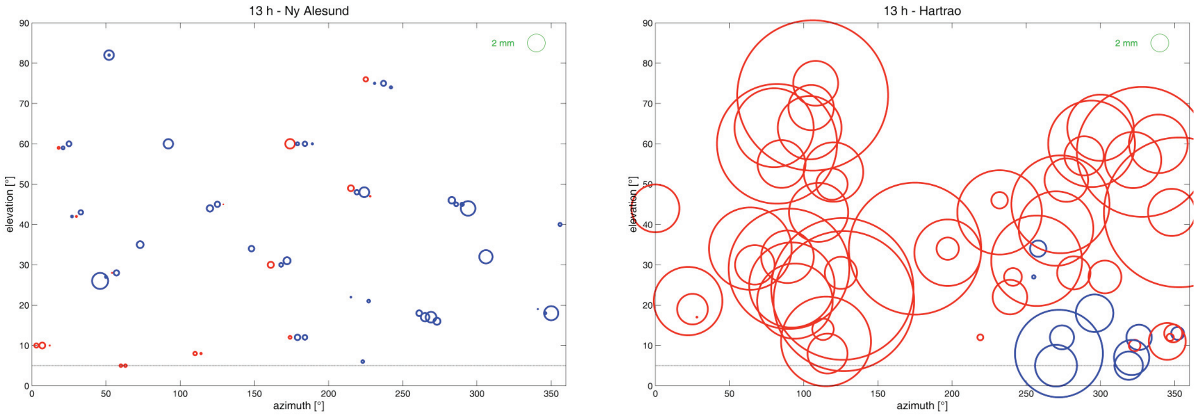

Fig 1 Bubble plots of wet zenith delay residuals for stations Ny Alesund (left plot) and HartRAO (right plot): The size of the bubbles corresponds to the absolute value of the residuals, the color indicates the sign (red: negative residuals, blue: positive residuals). The green circle corresponds to 2 mm and gives the scale.

HartRAO is in the Southern Hemisphere, not far from the equator and we expect the wet zenith delay to show a high variability above this station. This behavior is clearly reflected in the residuals. Furthermore the residuals for HartRAO show a significant systematic behavior with azimuth as well as with elevation. These systematics can be observed at many time epochs for nearly all stations.

To further investigate this effect, I had a closer look at the estimation for HartRAO at this time epoch. Figure 2 shows wzd in mm versus time in h. The simulated wzd is represented in dark blue, the estimation in green. Between time epochs 11.5 h and 13.3 h there are some huge peaks in the simulated data which significantly bias the estimation. The red dots in Figure 2 mark observations with elevation angles of 5-15 degrees. As can be seen, the large peaks in equivalent wzd time series are due to observations at low elevations.

The next step was to include the estimation of gradients as given in the IERS conventions 2003. The improvement of the estimation can be clearly seen in Figure 3. This plot shows the simulated wzd time series in dark blue, the estimation with the plf model in red and the

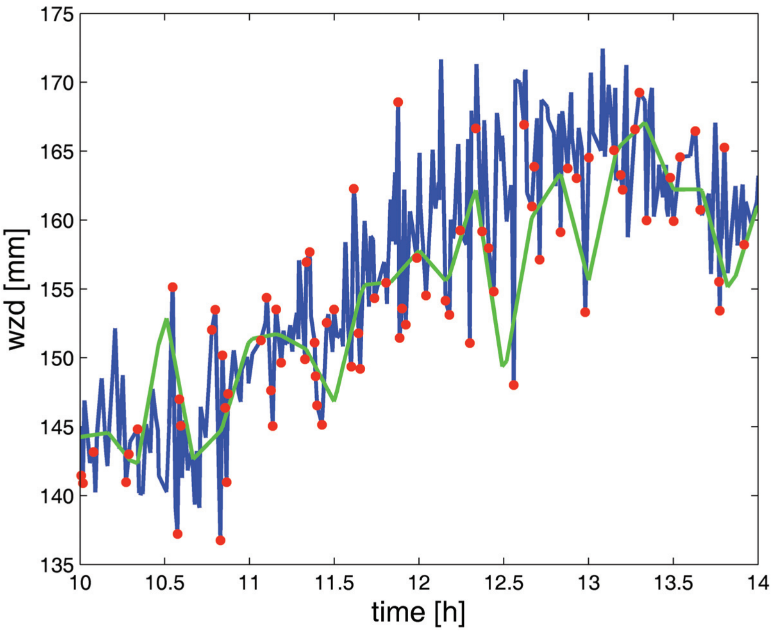

Fig 2 Biased estimation of wzd due to the large equivalent wzd of low observations. The blue line is the simulation, the green line indicates the estimation. Red dots mark observations below 15° elevation.

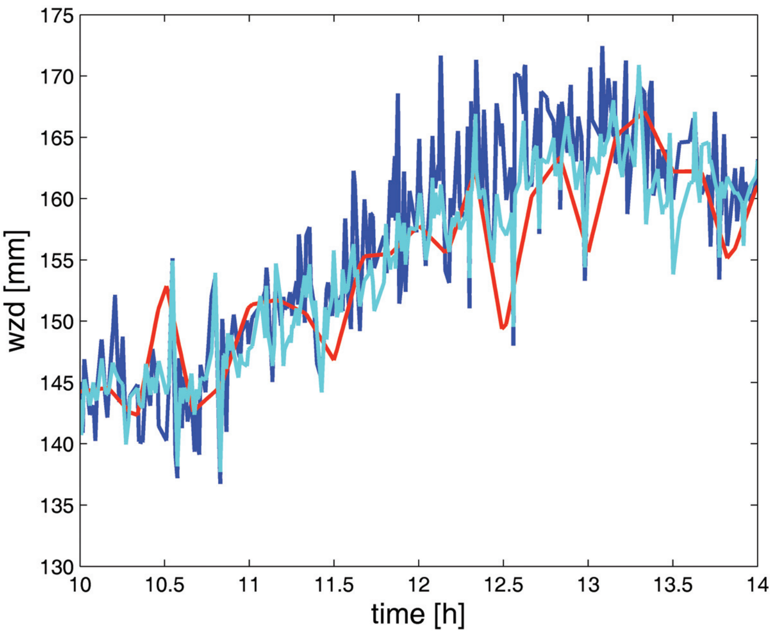

Fig 3 Comparison of the simple plf model and the gradient model. The blue line is the simulated wzd, the red line the estimation with the plf model and the cyan line the estimation with the gradient model.

estimation with the gradient model in cyan for station HartRAO. The time interval for the estimation was 10 min for the plf and 5 h for the gradients. The gradient model shows the ability to follow the fast variations of equivalent wet zenith delay pretty well. Especially the large peaks are modeled much better compared to the simple plf parametrization. Thus, the estimation is less biased. Nevertheless, there are still time epochs where the estimation does not fit well.

- The improvement in rms of wzd residuals due to the usage of the gradient model is dependent on the estimation interval of gradients: the improvement is 3 – 12 % and 11 – 24 % for estimation intervals of 5 and 1 h, respectively (compared to the rms of the simple plf model).
- Varying the time interval for the estimation of gradients also has a significant effect on the results. I tested time intervals of 5, 4, 2 and 1 h and computed the mean rms of wet zenith delay over 25 days. The improvement in residuals due to the shortening of estimation interval from 5 to 1 hours leads to an improvement of 8 – 14 %.
- It's necessary to point out that these values will not hold as soon as station clock and white noise are included.

3. Investigations on downweighting functions

As the estimation is biased due to observations at low elevations a downweighting of low observations would be obvious. Thus, I tested several downweighting functions. The wzd was modeled with plf (time interval: 10 min) and superimposed gradients (time interval was varied between 5, 4, 2 and 1 h). It's important to point out that the downweighting function was not only applied to observations below a specified elevation (e.g. 20° like in OCCAM) but to all used observations.
Again the computed wzd residuals are only due to modeling deficiencies!

The weighting functions tested were:

- 1/cos(e)**n with n = 1, 2, 3, 4
- sin(e)**n with n = 1, 2, 3
- 1/cot(e).

I have chosen 4 stations for these investigations: Ny Alesund with small residuals, HartRAO with large residuals and Westford and Wettzell with residuals somewhere between. For each station, I computed a mean rms of wet zenith delay residuals over 25 days.

Table 1 shows the improvement in rms of wzd residuals for the different downweighting functions in percent (compared to the rms without the application of downweighting). Time interval for the estimation of gradients was 5 h in this case:

Station	$\frac{1}{\cos(e)}$	$\frac{1}{\cos(e)^2}$	$\frac{1}{\cos(e)^3}$	$\frac{1}{\cos(e)^4}$	$\sin(e)$	$\sin(e)^2$	$\sin(e)^3$	$\frac{1}{\cot(e)}$
NYAL	4	9	13	14	16	22	20	19
HART	5	10	13	12	17	23	21	21
WETT	4	8	10	7	14	19	17	17
WEST	4	9	10	9	16	22	20	19

Table 1 Improvement in mean rms for different downweighting functions

The grade of improvement is station dependent as well as dependent on the time interval of gradient estimation. However, in all cases sin(e)**2 yielded the best results.

In Figure 4 I plotted the weighting functions 1/cos(e), 1/cos(e)**2, 1/cot(e) and sin(e)**2, the first three of which were normalized to be able to compare them with the sine. It can be seen that the 1/cos(e)**n functions assign high weights to only very high elevation angles.

The weighting functions tested here were chosen arbitrarily. For further investigations it would be good to use the behavior of wzd residuals for deriving the optimal weighting function.

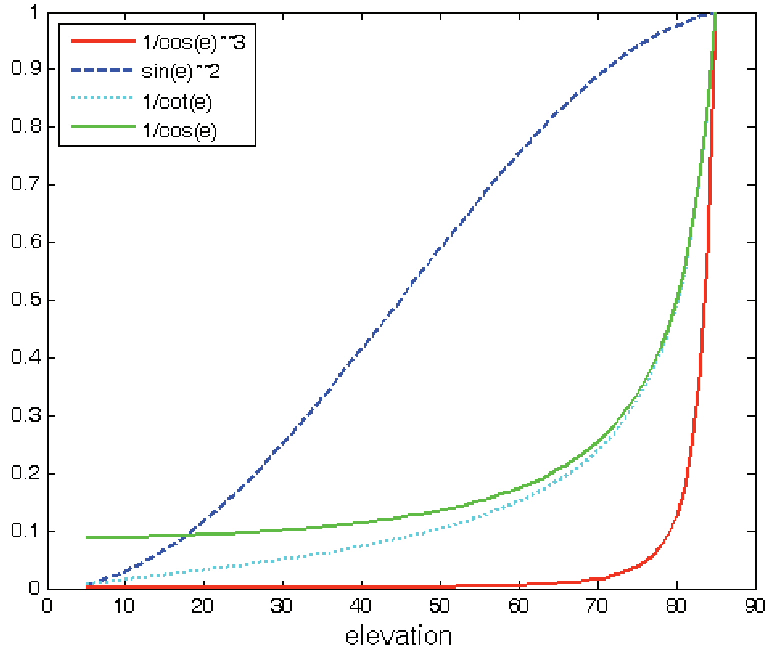

Fig 4 Functions for downweighting of low observations: 1/cos(e), 1/cos(e)**3 and 1/cot(e) were normalized to make them comparable to sin(e)**2.

4. Cutoff angle tests

As the above investigations have shown, that the low elevation observations tend to bias the estimation it might be worth thinking about increasing the elevation cutoff angle. Thus, investigations with this respect were carried out, too. For this purpose, the observations below a specific cutoff elevation were simply eliminated. Of course this means less observations. Nonetheless this does not mean running into numerical problems as the number of observations is fairly high. To overcome the decrease of observations it would be necessary to produce new schedules with higher elevation cutoffs and then simulate wzd time series for all participating stations.

The troposphere above a station, the station clock and the vertical component of station coordinates are highly correlated and can only be separated due to their distinct dependencies on zenith distance or elevation angle, respectively. As it is to suppose that observations at low elevations are needed in order to properly separate wzd, clock and station height, these parameters have all been taken into account in this investigation. The stochastic variations of station clock were simulated with a random walk corresponding to an ASD of $2\cdot10^{-15}$ @ 50 min and added together with the slant wet delay (turbulent equivalent wet zenith delay multiplied by a mapping function) and a white noise of 6 ps for each station being investigated. The wzd was modeled with plf (10 min) and superimposed gradients (1 h), the station clock with plf (1 h). The cutoff elevation angles tested were 5°, 7°, 10° and 15°. Station height was estimated once per day. The same four stations (Ny Alesund, HartRAO, Westford, Wettzell) were used as for the weighting function test. I did a run without the application of downweighting and a run where I applied sin(e)**2 as weighting function for the observations.

Figure 5 is a scatter plot of rms of wzd residuals for the 25 days of data. For Ny Alesund (left plot) it can be seen that increasing the cutoff angle does not yield any improvement. For HartRAO (right plot) there is a clear improvement when increasing the cutoff elevation from 5° to 15°, not only for the wzd but also for station clock and station height (not shown here).

Figure 6 shows scatter plots for station HartRAO. The left plot presents the results without the application of downweighting, the right plot presents the results obtained for a downweighting with sin(e)**2. It can be deduced that the scatter of rms is significantly lower when a downweighting function is applied.

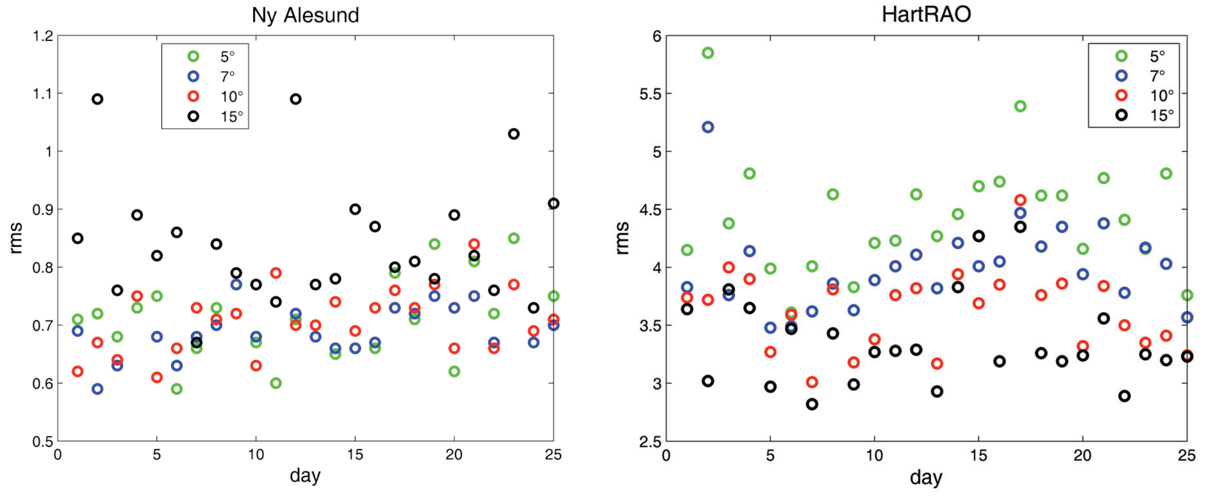

Fig 5 rms of wzd residuals for stations Ny Alesund and HartRAO for different cutoff elevation angles.

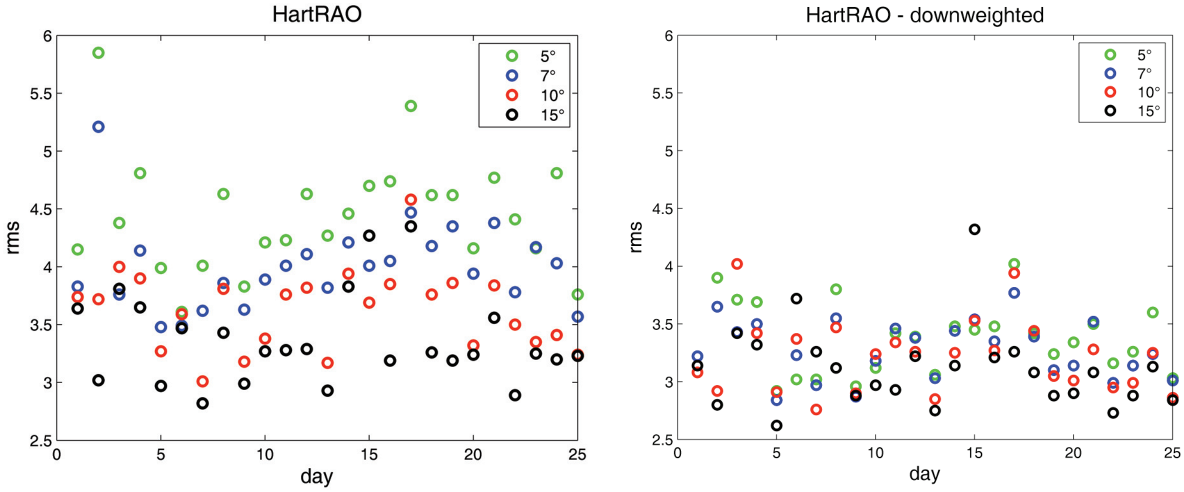

Fig 6 rms of wzd residuals for station HartRAO for different cutoff elevation angles. The left plot shows results without the application of a downweighting function, the right plot shows results for a downweighting with sin(e)**2.

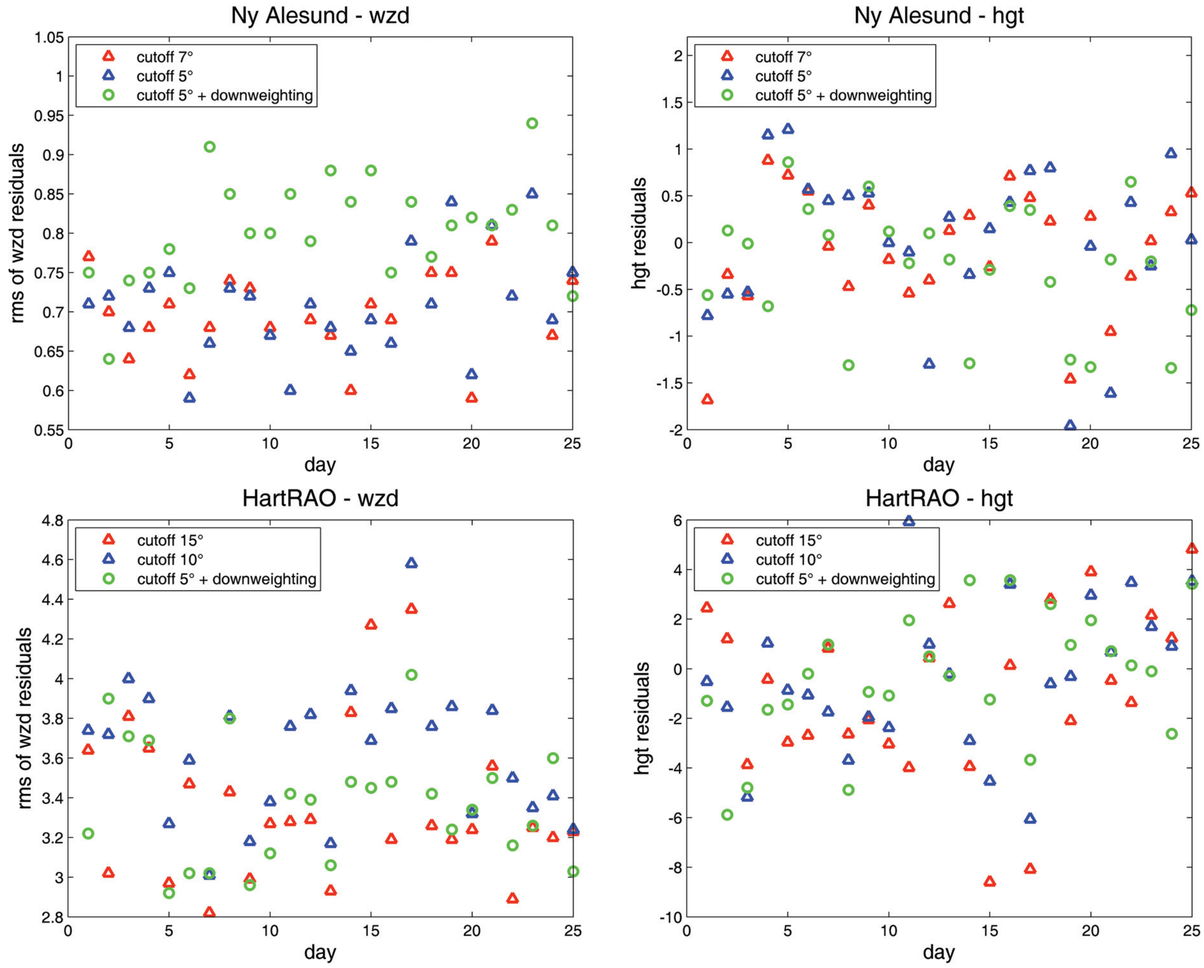

Fig 7 rms of wzd residuals (left) and height residuals (right) for stations Ny Alesund (upper plots) and HartRAO (lower plot). The green circles are for a cutoff angle of 5° where sin(e)**2 was applied as downweighting function. The blue and red triangles are for different cutoff angles (Ny Alesund: 5° (blue) and 7 ° (red), HartRAO: 10° (blue) and 15° (red)).

Figure 7 shows scatter plots for Ny Alesund and HartRAO. The left plots present rms of wzd residuals, the right plots station heights. The green circles are results for a cutoff angle of 5° where sin(e)**2 was applied as downweighting function. The blue and red triangles show results without the application of a downweighting function for different cutoff angles. For

Ny Alesund the best results could be obtained with cutoff angles of 5° (blue) and 7° (red). From the upper left plot in Figure 7 it can be seen that the application of a downweighting function yields worse results for the rms of wzd residuals compared to the results of an analysis where no downweighting was applied. In contrary, the height residuals (upper right plot) are improved (the scatter decreases). For HartRAO the application of the downweighting function yields improvements for both, rms of wzd and height residuals.

As wzd, station clock and station height are highly correlated and can only be separated due to their distinct dependencies on zenith distance, the correlations will increase when increasing the cutoff elevation angle. The same holds for the application of a downweighting function.

Figure 8 shows the correlations for station HartRAO. Wzd was modeled with plf (10 min) plus gradients (1 h), clock was modeled with plf (1 h), height was estimated once per day. In all three plots the dark blue (wzd-hgt), red (wzd-clk) and dark green (wzd-clk) lines are the correlations for an adjustment with a cutoff angle of 5° and no downweighting applied. Figure 8a compares these correlations to the correlations with the same cutoff angle but with sin(e)**2 applied as downweighting function. Figures 8b and 8c compare the correlations for different cutoff angles: 5° and 7° in Figure 8b and 5° and 15° in Figure 8c. It can be seen, that increasing the cutoff angle as well as applying a downweighting function increases the correlations between all three parameters, wzd, clk and hgt, where the grade of increase is different for all cases: For a cutoff angle of 7° or for applying downweighting the correlations wzd-hgt and wzd-clk are only slightly increased, whereas the increase is significantly larger for a cutoff angle of 15°. The correlations clk-hgt are least increased for a cutoff of 7°, most increased for a cutoff of 15° and somewhere between for the downweighting.

When looking at the residuals of wzd, clk and hgt for station HartRAO, it could be found, that a cutoff angle of 5 ° with the application of downweighting yielded best results for the rms of residuals (wzd and clk) and height residuals (the same holds for stations Westford and Wettzell, which are not shown here). The improvement (compared to cutoff 5° without downweighting) was approximately the same as for a cutoff angle of 15° without downweighting (see also lower plots in Figure 7), while increasing the correlations (especially wzd-hgt and wzd-clk) significantly less.

In conclusion of the cutoff angle tests it can be said:

- When the troposphere is dry, i.e. does not show a high variability, observations at low elevations are important for the separation of wzd, clk and hgt. In this case, increasing the cutoff angle makes things worse.
- When the variability of the troposphere is high, observations at low elevations have large equivalent wet zenith delays which tend to bias the estimation. In this case, increasing the cutoff angle improves the rms of wzd/clk residuals and height residuals. Of course this increases the correlations between the estimated parameters.

For dry stations, low observations are needed to obtain results as good as possible, while for wet stations low observations bias the estimation. It might therefore be a good compromise not to increase the cutoff angle but apply a weighting function for wet stations in the analysis. This yields an improvement of the residuals of estimated parameters while not so much increasing the correlations.

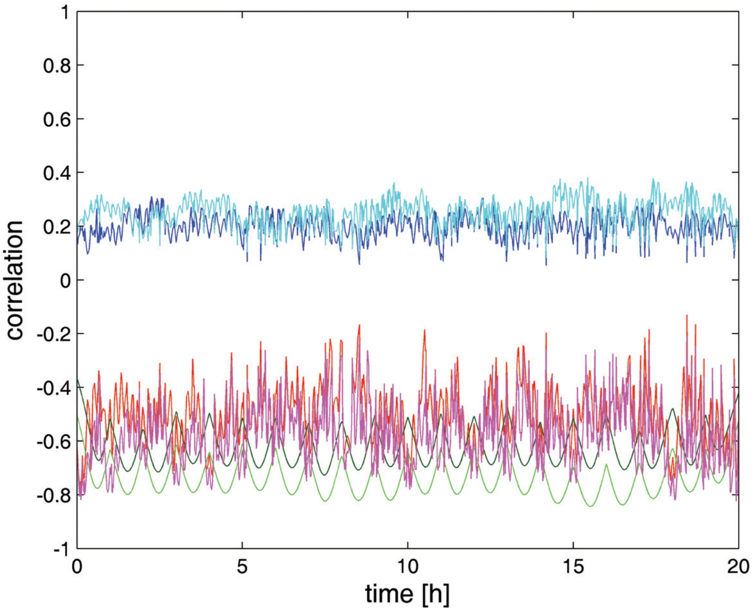

Fig 8a comparison of correlations. Blue lines: wzd-hgt, red lines: wzd-clk, green lines: clk-hgt. Dark blue, red and dark green lines: no downweighting applied, cyan, magenta and light green lines: sin(e)**2 applied for downweighting. Cutoff elevation is 5°.

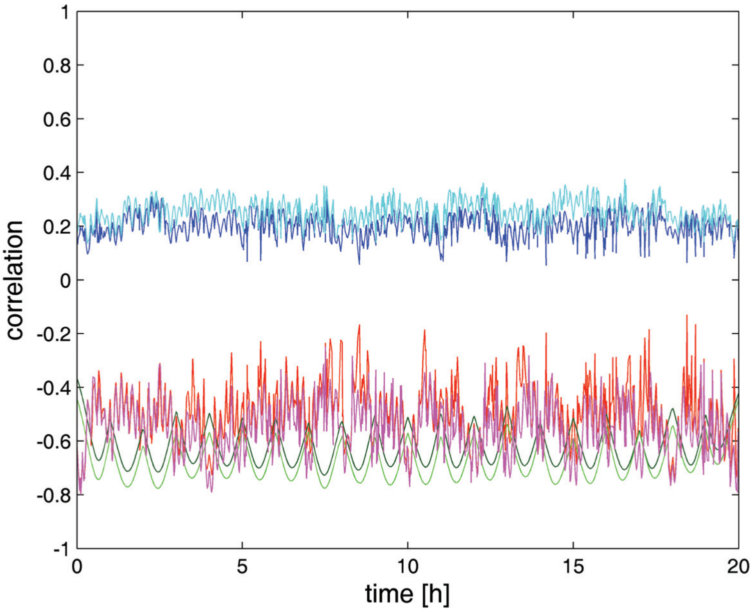

Fig 8b comparison of correlations. Blue lines: wzd-hgt, red lines: wzd-clk, green lines: clk-hgt. Dark blue, red and dark green lines: cutoff elevation 5°, cyan, magenta and light green lines: cutoff elevation 7°. No downweighting function applied.

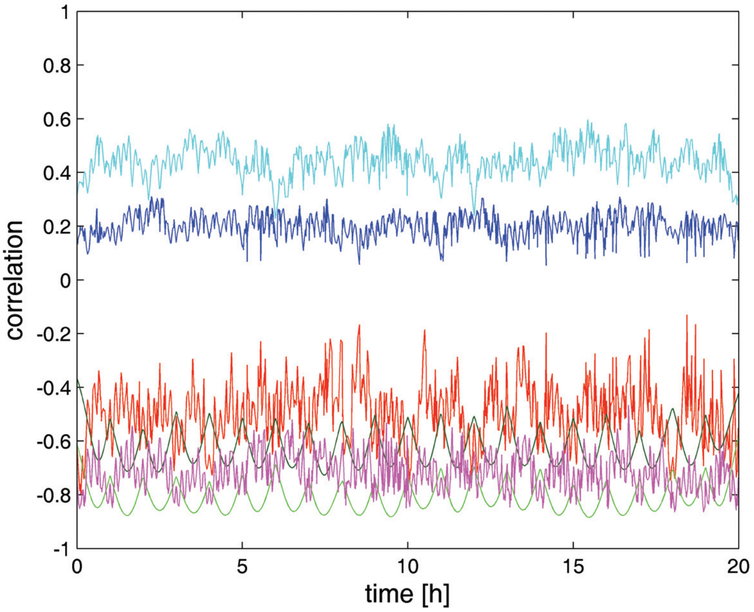

Fig 8c comparison of correlations. Blue lines: wzd-hgt, red lines: wzd-clk, green lines: clk-hgt. Dark blue, red and dark green lines: cutoff elevation 5°, cyan, magenta and light green lines: cutoff elevation 15°. No downweighting function applied.

5. Further Comments

The investigations carried out here were all performed with a MATLAB programme, only taking into account the troposphere, the station clock and white noise, only estimating wzd and clock parameters and station height and using an arbitrarily chosen weighting function. Not all conclusions will hold when using a VLBI analysis software and estimating additional parameters. It will therefore be necessary to

- deduce the optimal weighting function from investigations on the behavior of wzd residuals and
- perform extended investigations on the application of downweighting functions and cutoff anlge tests with OCCAM or Calc/Solve.

However, the results described above show that - assuming the turbulence model to be a realistic description of the tropopshere - the gradient model still has deficiencies and that low observations in a highly variable tropopsphere bias the estimation.